

--	--	--	--	--	--	--	--	--	--

First Semester B.E. Degree Examination, Dec.2019/Jan.2020 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. If $y = e^{-2x} \cos^3 x$, find y_n . (05 Marks)
 b. If $y = a \cos(\log x) + b \sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (06 Marks)
 c. Prove that the curves $r = a(1 + \sin \theta)$ and $r = a(1 - \sin \theta)$ cut orthogonally. (05 Marks)

OR

- 2 a. Find the radius of curvature of the curve $r^n = a^n \cos n\theta$. (05 Marks)
 b. Find the pedal equation of $r = 2(1 + \cos \theta)$. (06 Marks)
 c. If $y = e^{m \sin^{-1} x}$ prove that $(1-x^2)y_{n+2} - (2x+1)xy_{n+1} - (n^2+m^2)y_n = 0$. (05 Marks)

Module-2

- 3 a. Expand $\log \cos x$ in powers of $\left(x - \frac{\pi}{3}\right)$ using Taylor's series. (05 Marks)
 b. Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{1/x}$. (06 Marks)
 c. If $\sin u = \frac{x^2 y^2}{x+y}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3 \tan u$. (05 Marks)

OR

- 4 a. Using Maclaurin's series, expand $\log(1+e^x)$ in ascending powers of x . (05 Marks)
 b. If $u = f(x, y)$ and $x = r \cos \theta$, $y = r \sin \theta$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial u}{\partial \theta}\right)^2$. (06 Marks)
 c. If $u = x^2 + y^2 + z^2$, $v = x + y + z$, $w = xy + yz + zx$, evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (05 Marks)

Module-3

- 5 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$, determine the components of velocity and acceleration at $t = 1$ in the direction $2i + j + 2k$. (05 Marks)
 b. Find the directional derivatives of $\phi = x^2 yz + 4xz^2$ at the point $(1, -2, -1)$ along the direction of $2i - j - 2k$. (06 Marks)
 c. Prove that $\text{div}(\text{curl } \vec{F}) = 0$. (05 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

OR

- 6 a. If $\vec{F} = (3x^2 3yz)\hat{i} + (3y^2 - 3zx)\hat{j} + (3z^2 - 3xy)\hat{k}$, find (i) $\text{div } \vec{F}$ (ii) $\text{curl } \vec{F}$. (05 Marks)
 b. If $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ is irrotational, find a, b, c. (06 Marks)
 c. Prove that $\text{curl}(\phi A) = \phi(\text{curl } A) + \nabla\phi \times A$ (05 Marks)

Module-4

- 7 a. Find the reduction formula for $\int \sin^n x dx$ (05 Marks)
 b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$ (06 Marks)
 c. Evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}} dx$. (05 Marks)

OR

- 8 a. Find the orthogonal trajectory of the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is the parameter. (05 Marks)
 b. Solve $(1 + e^{x/y})dx + e^{x/y} \left(1 - \frac{x}{y}\right) dy = 0$. (06 Marks)
 c. A body in air at 25°C cools from 100°C to 75° in one minute. Find the temperature of the body at the end of three minutes. (05 Marks)

Module-5

- 9 a. Find the Rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & +1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$. (05 Marks)
 b. Apply Gauss-elimination method, to solve the system of equations $x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$. (06 Marks)
 c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to diagonal form. (05 Marks)

OR

- 10 a. Find the largest Eigen value and the corresponding Eigen vector of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and $X = (1 \ 0 \ 0)^T$ as initial vectors. (05 Marks)
 b. Solve the system of equations $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$. Carry out the 4th iterations, using Gauss-Seidal method. (06 Marks)
 c. Reduce the quadratic form of $x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz$ into canonical form. (05 Marks)
